Inequality in the Welfare Costs of Disinflation*

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Abstract

We use an incomplete markets economy to quantify the distribution of welfare gains and losses of the US "Volcker" disinflation. In the long run households prefer low inflation, but disinflation requires a transition period and a redistribution from net nominal borrowers to net nominal savers. Even with perfectly flexible prices, welfare costs may be significant for households with nominal liabilities. When calibrated to match the micro and macro moments of the early 1980s high inflation environment, almost *half* of all borrowers (14 percent of all households) would prefer to avoid the redistribution and equilibrium effects of the disinflation. This share depends negatively on the liquidity value of money and positively on the average duration of nominal borrowing.

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1 Introduction

In most environments, households would prefer an economy with low and stable inflation over one with persistently high inflation.[1] Even putting aside costs from relative price dispersion and inflation uncertainty, high inflation imposes resource costs as consumers alter their savings and consumption behavior in order to economize on nominal liquid assets subject to an inflation tax. By almost all measures, US households in the late 1970s faced a substantial inflation tax, with inflation in excess of 10 percent. These households would have preferred the low and stable price inflation now routine in the US. But, one cannot switch from one environment to another without some transition, or disinflation, period with its own benefits and costs. Paul Volcker, Chair of the Federal Reserve from 1979 to 1987, publicly committed to reducing inflation and largely succeeded by the mid 1980s. The period of tight monetary policy designed to tame high inflation became known as the "Volcker" disinflation.

We characterize one essential aspect of this disinflation, namely its unequal costs and benefits across households of varying types. The debate over the costs of disinflation typically centers on an aggregate sacrifice ratio, or the short-run loss in output or employment necessary to reduce the rate of price inflation. But the focus on aggregates necessarily abstracts from the underlying heterogeneous effects. We fix our sights on quantifying in a precise way the redistributive costs and benefits of this large disinflation.

These effects are potentially large. Households typically borrow in nominal contracts with long durations and hold a mix of real and nominal assets. A sudden decrease in inflation and inflation expectations increases the real burden of net nominal borrowers and redistributes these resources towards net nominal savers. In their seminal paper, Doepke and Schneider (2006a) carefully measure the distribution US households' net nominal liabilities, and they show the scope for disinflation-based redistribution is quantitatively significant. Their analysis considers the first order effects on the wealth distribution, but without an equilibrium model cannot on its own address welfare. Disinflation lowers the burden of the inflation tax whose incidence varies across households with heterogeneous money demand. Further, there are general equilibrium effects; a reduction the inflation tax encourages households to rebalance their portfolios towards money putting upward pressure on the equilibrium real interest rate. Both the lower inflation tax and the general equilibrium effect on the real interest rate are felt unevenly across the wealth and income distribution.

To isolate the welfare costs imposed by a sudden disinflation, we build a monetary economy with incomplete markets. We extend an Aiyagari (1994) economy to include money, valued for its liquidity services, and a durable good. Households face idiosyncratic earnings shocks as in a standard income fluctuation problem but now also face a portfolio choice problem. They must allocate resources between money, a long-term interest-bearing nominal asset and investment in

[1] Exceptions could include economies with seignorage financed transfers under incomplete markets where the insurance value of transfers to low income households might exceed losses from an inflation tax. Aiyagari (1990) provides an excellent summary of the costs and benefits in general equilibrium of reducing inflation.

durable goods. They also have access to secured borrowing against their durable stock in the form of a long-term nominal contract. This allows us to capture a common feature of household balance sheets: a fixed-rate mortgage secured by a house or other secured loans, such as those for automobiles. We consider an environment with perfectly flexible prices that adjust to the new inflation rate without any short-run output costs. While highly stylized, our purpose is to study the disinflation in the best case scenario in which there are no output losses from the monetary tightening. To the extent that, in practice, the required monetary tightening induces real effects on output through an upward sloping Phillips curve, we think of our results as conditioning out these aggregate effects.[2]

Then, we construct a *disinflation equilibrium path* as the economy's dynamic response to a sudden shift in the monetary policy stance. We calibrate the model to match key features of the US economy just before the Volcker disinflation, including 10 percent inflation. From this setting, the central bank announces a permanent shift to a lower inflation target of 3 percent. The disinflation policy is perfectly credible, so that with flexible prices inflation expectations immediately reflect the lower target. This of course differs from the experience during the Volcker disinflation, which was initially plagued with credibility problems. In that sense, as we describe, this is a best-case scenario.

The unanticipated shift in policy endogenously redistributes resources away from borrowers and towards savers. Borrowers deleverage in response to their unexpectedly large real debt burden. Savers further increase their savings to smoothly consume the real value of their windfall over future periods. The redistribution channel puts downward pressure on the real interest rate. At the same time, all households rebalance their portfolios as a reduced inflation tax substantially lowers the cost of holding real balances. The rebalancing restricts the aggregate supply of non-cash savings putting upward pressure on the real interest rate, a version of the Tobin (1965) effect.[3]

The full welfare effects of the disinflation will be determined jointly by the benefit of the lower inflation tax, the cost of the redistribution, and the equilibrium effects of the increase in the real interest rate. For savers, the welfare results are unambiguous. They receive a windfall from the redistribution, while also benefiting from the lower inflation tax and the higher real interest rate. For borrowers, the welfare benefit depends on whether the value of additional liquidity held following the lower inflation tax is enough to compensate them for both the wealth lost in the redistribution and the increase in the real interest rate. In our baseline calibration, 14 percent of households prefer to remain in the high inflation steady state rather than face the costs of the redistribution despite the fact that 48 percent are nominal borrowers. This suggests that the benefit of the lower inflation tax is enough to compensate most borrowers for the increase in their real debt burden and the increase in the real interest rate.

However, the costs are borne unequally across the income distribution. The average household

[2]It is of course true, that the aggregate real effects may themselves be distributed unequally. See for example Krueger, Mitman, and Perri (2016), who examine the heterogeneous effects of an aggregate technology shock.

[3]This effect is attenuated by the elastic demand for capital in our production economy, which relieves some of the pressure on the real interest rate.

needs to be compensated 2.6 percent of their wealth to avoid the disinflation, while this compensation needs to be as high as 120 percent for low income households. In a straight up or down vote, 44 percent of low income households would choose to live with high inflation despite the high inflation tax and almost universally preferring the low inflation economy in the long run. A small share of low-income borrowers prefer high inflation even in the long run because of its downward equilibrium effects on the real interest rate. We ask how these welfare results depend quantitatively on the burden of the inflation tax, which is determined by the liquidity value of money. We compare our baseline welfare results to a calibration of a cashless limit economy. Without any offsetting gains from the reduction of the inflation tax, all borrower households are sufficiently hurt by the redistribution that they would prefer to remain in the high inflation steady state.

We then compare the welfare results from the baseline calibration to a version with a shorter duration for nominal borrowing, which decreases the size of the redistribution. In our baseline calibration, the duration of nominal assets is 4.5 years. We adjust the duration so that households borrow using one-period nominal bonds. Because the redistribution is smaller, borrowers are more willing to face the burden of the redistribution in exchange for the benefit of the lower inflation tax. In this economy only a small fraction would vote to remain in the high inflation steady state, less than half of those that would vote for high inflation in the baseline calibration. The differences between the baseline calibration and the alternative experiments highlight the importance of capturing all three channels (redistribution, decrease in the inflation tax, and increase in the real interest rate) when considering the welfare costs of a disinflation.

Finally, in light of persistently low levels for the natural rate of interest and concerns about a low inflation target leaving monetary policy close to the zero lower bound on nominal interest rates (Ball, 2014; Coibion, Gorodnichenko, and Wieland, 2012), we apply our framework to consider an *increase* in the inflation target. The debate about raising the inflation target generally ignores the distributional effects of transitioning to higher inflation. As in the case of the disinflation, we find the costs and benefits of an inflationary period are unevenly distributed across the income and wealth distributions. Despite the fact that 48 percent of households hold nominal debt and will benefit from the redistribution, 88 percent of households prefer to stay in the low inflation environment rather than face the higher inflation tax. However, 48 percent of low income households would prefer to go through the inflation despite facing a higher inflation tax, while less than 2 percent of high income households prefer the transition to high inflation. This is because low income households benefit from the devaluing of their nominal debt and a slightly lower real interest rate. High income households, on the other hand, are savers and therefore are hurt by the redistribution, the lower real interest rate, and the higher inflation tax.

After discussing the previous literature, in Section 2 we describe our model and in Section 3 we describe the data and model calibration. In Section 4 we describe the transition period for our baseline experiment: a surprise disinflation from 10 to 3 percent, evoking the Volcker disinflation, and discuss the welfare effects of the disinflation. Section 5 compares the results from our baseline calibration to a cashless economy and an economy with only short duration bonds, and briefly considers the welfare costs of raising the inflation target. Section 6 concludes.

Contribution to the Literature

Our analysis of the Volcker disinflation highlights the importance of considering three channels to understand the welfare costs of a permanent change in the steady state inflation rate: the revaluation of nominal assets, the change in the inflation tax, and the equilibrium effects on the real interest rate from portfolio rebalancing. We discuss the literature on each of these three channels below. A key contribution of our work is that we are the first to consider this question in a model with a rich enough portfolio choice problem on the household side to jointly consider all three channels.

We include money explicitly to allow for long-run benefits of reducing steady state inflation. With money, inflation serves as a tax, which directly effects households demand for real balances and indirectly effects them through the aggregate effect on the real interest rate. The idea of inflation as a consumption tax is well established, and the effects can be significant for welfare.[4] Allais, Algan, Challe, and Ragot (2016) and Cao, Meh, Ríos-Rull, and Terajima (2018) consider the welfare consequences of an inflation tax in models with incomplete markets, but they do not examine the redistribution consequences of a change in inflation along the transition path. The long run equilibrium effects of inflation on the real interest rate are developed in Dotsey and Ireland (1996) and Aiyagari et al. (1998). They introduce a channel where inflation draws resources away from production and into credit services to avoid an inflation tax, inducing a general equilibrium effect on the real interest rate.

The redistribution or Fisher channel has been most recently studied in work by Auclert (2015) and Doepke and Schneider (2006a). They show that an unexpected shock to the inflation rate will revalue nominal assets causing a redistribution between borrowers and savers. Doepke and Schneider (2006a) reinvigorated an early literature on inflation and redistribution (see for example Bach and Stephenson, 1974) by documenting the economically significant net nominal exposure of various cohorts and sectors in the US economy and conducting a reduced form calculation of the redistribution from a surprise inflation episode.

Several papers have also examined the effect of the Fisher channel quantitatively in a heterogeneous agent model with incomplete markets, but they exclude the long-run benefits of lower inflation. Doepke and Schneider (2006b) and Meh, Ríos-Rull, and Terajima (2010) do this by treating a surprise an inflation as an exogenous redistribution of wealth. However, instead of modeling inflation explicitly, they start with a stationary distribution of wealth, revalue nominal assets and examine the transition path. Instead, we model inflation directly and consider a portfolio choice problem by households that captures the welfare long-run welfare effects of a lower inflation

[4] While Lucas (2000) and Bailey (1956) find small estimates of the welfare costs of inflation from integrating under an estimated money demand curve in a representative agent economy. Chatterjee and Corbae (1992) and Imrohoroglu (1992) find that incomplete market arrangements can significantly amplify welfare costs of an inflation tax over the earlier complete markets estimates. Attanasio et al. (2002) and Erosa and Ventura (2002) also show that transaction costs from inflation vary considerably across households.

tax. Without the inflation tax, changing the steady state rate of inflation amounts to a simple redistribution of wealth with no long-run benefits.

Our project also relates to recent work that considers the transmission of monetary policy in an incomplete markets framework.[5] However, the goal of this literature is distinct from our own. These papers are interested in characterizing how heterogeneity will affect the central bank's ability to use monetary policy as a tool to counteract short term business cycle fluctuations. In contrast, we are interested in characterizing the redistribution effects of a long run change in central bank policy. Kaplan, Moll, and Violante (2018), Gornemann, Kuester, and Nakajima (2016) and Mitman, Manovskii, and Hagedorn (2017) extend these models to include a New Keynesian block on the production side. However, their focus is also on the transmission of monetary policy rather than the distributional consequences of a large permanent shock to steady state inflation. An interesting exception is Sterk and Tenreyro (2018) who use an incomplete markets model with money to consider the effect of a redistribution between households and government on the pass-through of monetary policy.

Finally, our project relates to a recent debate on whether central banks should increase the steady state rate of inflation in order to avoid hitting the zero lower bound (Ball, 2014; Coibion, Gorodnichenko, and Wieland, 2012). While we do not directly consider the welfare benefits of avoiding the zero lower bound, we use our framework to consider the welfare costs of raising the inflation target, including both the long-run welfare costs of the higher inflation tax and the costs along the transition path from the redistribution. Central banks should weigh these costs against the benefit of avoiding the zero lower bound when considering a high inflation target.

2 Monetary economy with heterogeneity

We start by extending an Aiyagari (1994) economy to include money, durable goods, and long-term secured lending contracts. As before, households cannot perfectly insure idiosyncratic shocks to their labor productivity, but now may trade in cash, a long-term nominal asset and durable goods, of which the latter also serves as collateral. We first consider a stationary environment with high inflation. To study the welfare effects of a disinflation, we quantify the response of this stationary economy with high inflation to an unanticipated change in the monetary policy stance of the government. Since prices are perfectly flexible, inflation adjusts immediately to its new permanently lower level, but the economy is no longer stationary. It converges in finite time to a new stationary distribution with low inflation. We can thenmeasure both the short-run and long-run benefits and costs of inflation across the evolving distribution of households along the equilibrium path.

[5]For example, Wong (2015), Cloyne, Ferreira, and Surico (2019), Garriga, Kydland, and Šustek (2017) and Ozkan, Mitman, Karahan, and Hedlund (2017) show that part of the consumption response to a monetary policy or inflation shock will take place through the refinancing of household debt or the effect of interest rate changes on households with adjustable rate mortgages.

2.1 Preliminaries

Time is discrete. The economy consists of a large number of dynastic households indexed by i and represented by the unit interval $i \in [0,1]$. Each supplies labor inelastically to a single production sector; a government implements fiscal and monetary policy.

This is a monetary economy where money $\tilde{m}$ is together a numeraire, a store of value and a source of liquidity services to the households. As numeraire we define the money price of period t output as P_t and denote the real value of money balances as $m \equiv \tilde{m}/P$. Throughout, we use the $\tilde{}$ notation to indicate a nominal variable. We capture the liquidity value of money by including real balances m directly in the household's preferences, although the economy would be little changed if demand for real balances were instead determined by shopping time or cash-in-advance constraints.[6]

Preferences and endowments. Households have identical preferences over sequences of non durable consumption, c_t, real balances, m_t, and the service flow from durables, d_{t-1}, ordered by

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t u \left(c_t, m_t, d_{t-1} \right) \right], \tag{1}$$

with discount factor β and standard assumptions on u. The expectation is over only the household's idiosyncratic labor efficiency; there is no aggregate uncertainty. Each household has one unit of raw labor that it supplies inelastically to a production sector. Its efficiency $z_t \in \{z_1, \ldots, z_{N_e}\}$ follows a Markov chain with constant transition matrix $P = [p_{lk}]$ initialized from its stationary distribution $\bar{p} \in \mathbb{R}^{N_z}$. Since draws are independent across households, a law of large numbers implies that the aggregate quantity of efficiency units of labor N is constant and equal to $E[z_t]$.

Production The production sector consists of a representative firm that uses the installed capital stock, K_{t-1}, and efficiency units of labor, N, to produce output with a stationary constant returns to scale technology

$$Y_t = F \left(K_{t-1}, N \right).$$

Throughout, we use capital letters to denote aggregate quantities. Aggregate output, Y_t, may consumed by households, C_t, invested, either in the capital stock, I_t^K, or durables, I_t^D, or purchased by the government, G_t, so that the aggregate resource constraint is

$$Y_t = C_t + I_t^K + I_t^D + G_t. \tag{2}$$

Given aggregate investment, I_t^K, the capital stock depreciates at rate δ^K and follows the law of motion

$$K_t = \left(1 - \delta^K \right) K_{t-1} + I_t^K. \tag{3}$$

[6]With some small alterations to the timing assumptions our model would be equivalent to cash-credit or shopping time microfoundations of money demand. We would expect similar results in any model where inflation generates utility or resources costs to economizing on liquid assets.

Durable investment may vary across households. Given aggregate durable investment, $I_t^D = \int_0^1 i_{it}^D di$, the aggregate durable stock follows the law of motion

$$D_t = \left(1 - \delta^D\right) D_{t-1} + I_t^D + \int_0^1 \Psi_i di, \tag{4}$$

The last term sums the household-level durable adjustment cost, $\Psi\left(d_{it}, d_{it-1}\right)$ across households so the aggregate adjustment cost will depend on the underlying distribution of durable investment across households.

2.2 Market arrangements

There are competitive labor and capital rental markets with prices $P_t W_t$ (per efficiency unit) and $P_t V_t$, respectively. Financial intermediation is through a representative mutual fund, which owns the capital stock and makes secured long-term *nominal* lending contracts to borrower households.

Household borrowing and saving. We distinguish borrower and saver households by their nominal financial net worth, $\tilde{a}$, and further assume that they cannot simultaneously borrow and save.[7] Saver households, $\tilde{a} \geq 0$, hold equity in the mutual fund. Borrower households, $\tilde{a} < 0$, borrow from the mutual fund. These loans may be used to purchase durable goods, which are pledged as security. There is no unsecured borrowing.

To match the higher duration borrowing observed for U.S. households, we build on Hatchondo and Martinez (2009) and Auclert (2015) and allow the mutual fund to offer long-term secured nominal debt contracts with a duration that depends on parameter ρ. A household who borrows $P_t^{\tilde{L}} \tilde{l}_t$ towards a purchase of durable goods agrees to a perpetual stream of payments $\tilde{l}_t,\ \tilde{l}_t\rho,\ \tilde{l}_t\rho^2, \ldots$ that decay at rate ρ, where $P_t^{\tilde{L}}$ is the price of the loan. With $\rho = 0$ this is a one period loan, but increasing ρ stretches the duration of the loan, mimicking longer-term borrowing such as mortgages. With borrowing $\tilde{a} = -P_t^{\tilde{L}} \tilde{l}_t$, the secured lending constraint,

$$-\frac{\tilde{a}}{P_t^{\tilde{L}}} \left(1 + \rho P_{t+1}^{\tilde{L}}\right) \leq \mu \left(1 - \delta^D\right) d_t P_{t+1}, \tag{5}$$

ensures that value of the security can be used to repay the loan. The left hand side is the nominal value of the household's borrowing in period $t+1$, and the right hand side is some fraction $\mu \leq 1$ times the nominal value of the households remaining durables in period $t+1$.[8]

Mutual fund. Saver households purchase equity, $\tilde{E}_t$, in the mutual fund, which the fund invests in capital, $P_t K_t$, and in secured long-term lending $P_t^{\tilde{L}} \tilde{L}_t$. Capital purchased at price P_t is rented

[7] It is not sufficient to simply track net financial assets. While with equivalent rates of return households would be indifferent across a continuum of gross borrowing and saving positions given the same net financial asset position, an unanticipated disinflation would affect the ex-post return on borrowing and saving differently.

[8] Since the borrowing constraint (5) depends on future prices, an unexpected disinflation may push constrained households beyond their secured borrowing limits ex-post. For this reason, we set μ low enough so the value of the loan will never exceed the full value of the remaining durables.

to firms the following period at price $P_{t+1}V_{t+1}$. Including the value of the undepreciated capital, the gross nominal return on capital investment is

$$R_{t+1}^{K} = \frac{V_{t+1}P_{t+1} + (1-\delta)\,P_{t+1}}{P_t}.$$

Given the structure of the long-term debt contract, the the gross nominal return on the fund's investment in long-term lending is

$$R_{t+1}^{\tilde{L}} = \frac{1+\rho P_{t+1}^{\tilde{L}}}{P_t^{\tilde{L}}}.$$

Increases in ρ increase the duration of the loan portfolio and thus the sensitivity of its value to the nominal interest rate.[9] We let ϕ_t denote the share of the fund's assets (and equity) held in loans

$$\phi_t = \frac{P_t^{\tilde{L}}\tilde{L}_t}{P_t^{\tilde{L}}\tilde{L}_t + P_tK_t}.$$

Of course since the mutual fund is financed entirely by equity, total equity equals total assets, $\tilde{E}_t = P_t^{\tilde{L}}\tilde{L}_t + P_tK_t$. Because lending is a nominal asset and capital is a real asset ϕ_t is also the nominal share of assets. Then, the gross nominal return on mutual fund equity can be written as

$$R_{t+1}^{\tilde{E}} = \phi_t\frac{1+\rho P_{t+1}^{\tilde{L}}}{P_t^{\tilde{L}}} + (1-\phi_t)\,\Pi_{t+1}\left(V_{t+1}+1-\delta\right), \tag{6}$$

where $\Pi_{t+1} = P_{t+1}/P_t$ is the gross inflation rate, which converts the real return on capital to a nominal return.

We consider an equilibrium where the mutual fund holds both capital and loans in its portfolio, $\phi_t \in (0,1)$, thus the expected return on bank equity must equal the expected return on its loan and capital investments.

$$E_t\left[R_{t+1}^{\tilde{E}}\right] = \frac{1+\rho E_t\left[P_{t+1}^{\tilde{L}}\right]}{P_t^{\tilde{L}}} = E_t\left[\Pi_{t+1}\right]\left(V_{t+1}+1-\delta\right). \tag{7}$$

With no fluctuations in aggregate productivity and a constant labor supply, the future rental rate V_{t+1} is measurable in period t. Under perfect foresight, prices $P_{t+1}^{\tilde{L}}$ and inflation Π_{t+1} the expectation operator may appear unnecessary. However, we write them this way to emphasize how if prices were to differ from their expected values because of a completely unanticipated disinflation,

[9] For example, with a constant gross nominal return R, in equilibrium $P_t^{\tilde{L}} = \frac{1}{R-\rho}$ and the duration of the lending portfolio would be $\frac{R}{R-\rho}$. To see this, note that the Macauly duration is defined as the horizon weighted average of discounted future cash flows relative to the price. In this case:

$$D \equiv \sum_{k=1}^{\infty} k\left(\frac{1}{R}\right)^{k}\rho^{k-1}/P_t^{\tilde{L}} = \frac{R-\rho}{\rho}\sum_{k=1}^{\infty} k\left(\frac{\rho}{R}\right)^{k} = \frac{R}{R-\rho}.$$

The third equality follows from the convergent power series $\sum_{k=1}^{\infty} kz^k = z/\left(1-z\right)^2$.

the ex-post return on equity would be different than its expected return.

The modeling choice of equity over debt financing for the intermediary is important in this case. A lower than expected realization of inflation reduces the nominal return on capital, and if $\rho > 0$, increases the nominal return on lending. The ex-post return on mutual fund equity simply adjusts according to (6). If the intermediary were instead financed by debt, such as a bank deposits, the increases in the nominal return on long-term lending might not offset the reduction in the nominal return on capital and the intermediary would not be able to pay the interest owed on deposits. If the return to paid to deposits matched the return on lending, insolvency would be assured.

2.3 Household behavior

Given these arrangements, the timing is as follows. In each period t, a household begins with its nominal savings $\tilde{m}_{t-1} + \tilde{a}_{t-1}$ and its remaining durables $\left(1 - \delta^D\right) d_{t-1}$. The current price level P_t is realized. Households earn a nominal wage $P_t W_t$ per efficiency unit of labor. They may purchase consumption c_t and invest in durable goods i_t^D both at price P_t. They can also adjust their money holdings $\tilde{m}_t$ and borrow $\tilde{a}_t = P_t^{\tilde{L}} \tilde{l}_t$ in loans $\tilde{l}_t$ sold at price $P_t^{\tilde{L}}$ subject to borrowing constraint (5) or save through the purchase of equity, $\tilde{a}_t = \tilde{e}_t$ (but not both). Using (4) to substitute for i_t^D and dividing by P_t to express in terms of output, the household is subject to a sequence of real budget constraints

$$c_t + m_t + P_t^{\tilde{L}} l_t + e_t + d_t + \Psi\left(d_t, d_{t-1}\right) = z_t W_t + \frac{a_{t-1}(1 + \mathbf{1}_{\{a_{t-1}<0\}} \rho P_t^{\tilde{L}}) + m_{t-1}}{\Pi_t} + (1 - \delta^D) d_{t-1} \quad (8)$$

where net financial assets $a_t = e_t - P_t^{\tilde{L}} l_t$, but because the household cannot simultaneously borrow and save $l_t e_t = 0$.

Given initial real savings $a_{-1} + m_{-1}$ and durable stock d_{-1}, each household maximizes (1) subject to sequences of borrowing (5) and budget (8) constraints for $t \geq 0$. To characterize household behavior, it is helpful to rewrite its sequence problem recursively. We first define the household's real net worth in period t

$$q_t = \frac{m_{t-1} + a_{t-1}(1 + \mathbf{1}_{\{a_{t-1}<0\}} P_t^{\tilde{L}} \rho) + e_{t-1}}{\Pi_t} + \left(1 - \delta^D\right) d_{t-1}, \quad (9)$$

which is measured after inflation Π_t is realized. For all $t \geq 0$, given real net worth q_t, accumulated durables d_{t-1}, and labor efficiency z_t we let $V_t\left(q_t, d_{t-1}, z_t\right)$ denote the value of a household in period t. Then for all $t \geq 0$ V_t satisfies a sequence of Bellman equations

$$V_t\left(q_t, d_{t-1}, z_{it}\right) = \max_{c_t, m_t, d_t} \left\{U\left(c_t, m_t, d_{t-1}\right) + \beta E\left[V_{t+1}\left(q_{t+1}, d_t, z_{t+1}\right) | z_t\right]\right\}, \quad (10)$$

each subject to a real budget constraint

$$c_t + a_t + d_t + \Psi\left(d_t, d_{t-1}\right) + m_t = q_t + z_t\left(1 - \tau_t\right) W_t,$$

and real borrowing constraint

$$-\frac{a_t}{P_t^{\tilde{L}}}(1+P_t^{\tilde{L}}\rho) \leq \Pi_{t+1}\left(b+\mu\left(1-\delta^D\right)d_t\right).$$

Next period q_{t+1} is determined according to (9). The value functions depend on t through interest rates i_t, wages W_t and fiscal and monetary policy, which may vary over time. We abuse notation slightly and label the policy functions that satisfy the Bellman equation as $c_t\left(q_t, d_{t-1}, z_t\right)$, $m_t\left(q_t, d_{t-1}, z_t\right)$, $a_t\left(q_t, d_{t-1}, z_t\right)$, $e_t(q_t, d_{t-1}, z_t)$ and $d_t\left(q_t, d_{t-1}, z_t\right)$.

2.4 Firm behavior

The production sector is straightforward: a representative firm rents capital and efficiency units of labor in a competitive market at real prices V_t and W_t respectively. Imposing market clearing in the labor market, profit maximization requires

$$F_k\left(K_{t-1}, N\right) = V_t; \quad F_n\left(K_{t-1}, N\right) = W_t. \tag{11}$$

The production sector is deliberately simple. Its primary purpose is to generate an elastic demand for savings that will limit the fluctuations in the real interest rate from shifts in the aggregate supply of savings.

2.5 Government

Also for simplicity, the government is a consolidated fiscal and monetary authority. It adjusts the nominal money stock to achieve an inflation target and uses the seignorage revenue and tax revenue to finance government expenditures determined by a balanced budget constraint. Specifically, given an inflation target Π^*, and initial nominal liabilities $\tilde{M}_{-1}$, *a fiscal and monetary policy* is a sequence of money stocks $\left\{\tilde{M}_t\right\}$ and government expenditures $\{G_t\}$ that implement the inflation target $P_t/P_{t-1} = \Pi^*$ and satisfy

$$P_t G_t = \tilde{M}_t - \tilde{M}_{t-1} + \int \tau_t z_{ti} W_t di. \tag{12}$$

the government balanced budget constraint (12) for all $t \geq 0$.[10]

With incomplete markets, passive fiscal policy, even with lump sum transfers, is non Ricardian, and the welfare costs depend on the details of the fiscal backing of the monetary policy regime. This dependence is both because aggregate savings depends nonlinearly on savings and because lump sum transfers provide some insurance by reducing the variance of household income. We have the government directly adjust spending instead of transfers to avoid capturing these effects on welfare from changes in the proceeds of the inflation tax.

[10]We think of government expenditures as completely separable from household preferences over consumption and real balances.

2.6 Aggregating over heterogeneous households

Before characterizing an equilibrium, we first define a measure to keep track of the distribution of households. Let $\psi_t(q,d,z)$ be the measure of households that *begin* period t with $q_t \le q$, $d_{t-1} \le d$ and efficiency $z_t = z$. For $t \ge 0$, given household policy rules a_t, m_t and d_t, this measure must satisfy the law of motion:

$$\psi_t\left(q',d',z_j\right) = \sum_{k=1}^{N_z}\left[\iint \mathbf{1}\left\{\left(\frac{a_{t-1}(q,d,z_k)\left(1+\mathbf{1}_{\{a_{t-1}<0\}}P_t^{\tilde{L}}\rho\right)+m_{t-1}(q,d,z_k)}{\Pi_t}+(1-\delta_D)d_{t-1}(q,d,z_k)\right)\right.\right.$$
$$\left.\left.\le q' \bigcap d_{t-1}(q,d,z_k) \le d'\right\}\psi_{t-1}(\partial q,\partial d,e_k)\right]p_{kj}. \tag{13}$$

This captures the evolution of real net worth and durables given each household's choices and the realization of the Markov state z. The dependence on t is through the household policy rules, which are themselves functions of equilibrium prices.

Using ψ_t, we can define aggregate quantities for consumption

$$C_t \equiv \sum_{k=1}^{N_z}\left[\iint c_t(q,d,z_k)\,\psi_t(\partial q,\partial d,z_k)\right]\bar{p}_k,$$

demand for real balances,

$$M_t^d \equiv \sum_{k=1}^{N_z}\left[\iint m_t(q,d,z_k)\,\psi_t(\partial q,\partial d,z_k)\right]\bar{p}_k, \tag{14}$$

durables,

$$D_t \equiv \sum_{k=1}^{N_z}\left[\iint d_t(q,d,z_k)\,\psi_t(\partial q,\partial d,z_k)\right]\bar{p}_k,$$

and aggregate savings,

$$S_t \equiv \sum_{k=1}^{N_z}\left[\iint a_t(q,d,z_k)\left(1+\mathbf{1}_{\{a_t<0\}}\rho P_t^{\tilde{L}}\right)\psi_t(\partial q,\partial d,z_k)\right]\bar{p}_k. \tag{15}$$

The outer sum in each definition is over the distribution of z, which is stationary.[11]

2.7 Stationary high inflation equilibrium

We define the stationary *high inflation equilibrium* as follows. Given a fiscal and monetary policy with inflation target Π^H, a stationary high inflation equilibrium is

1. Prices $P_t^{\tilde{L}}$, $P^{\tilde{e}}$, W, V that grow at a constant inflation rate inflation, $\Pi = \Pi^H$, that satisfy

[11] Recall that the transition matrix of the Markov chain for labor efficiency z is defined $P = [p_{lj}]$ and the chain is initialized from its unique ergodic distribution $\bar{p} \in \mathbb{R}^{N_e}$.

the no arbitrage condition (7) and profit maximization (11).

2. A stationary value function $V(q,d,z)$ that solves the Bellman equation (10) with decision rules $c(q,d,z)$, $m(q,d,z)$, $d(q,d,z)$, and $a(q,d,z)$.

3. A stationary measure ψ^H that satisfies (13) given household decision rules.

4. Aggregate capital demand from (11) equaling aggregate savings from (15)

$$K = S. \tag{16}$$

5. The government budget constraint (12) holds with $M = M^d$ given aggregate demand for real balances (14).

2.8 Disinflation equilibrium path

We use the stationary high inflation equilibrium as the starting point for the following experiment. What if, in a high inflation stationary equilibrium at $t = 0$, in the following period $t = 1$, the government abruptly changes its monetary policy stance? We consider a scenario where it abandons its original inflation target Π^H and makes a credible commitment to a lower inflation target $\Pi^L < \Pi^H$ for $t \geq 1$. The announcement takes households by surprise as they have already made their portfolio choices in period $t = 0$ in the high inflation equilibrium. Now, in period $t \geq 1$, the government does whatever it takes to reach the lower inflation rate by accommodating the initial shock to aggregate real money demand as households rebalance their portfolios under the new lower inflation environment.

Redistribution. The change in the realized level of inflation $\Pi_1 = \Pi^L$ from its previously anticipated perfect foresight value Π^H alters the real value of household net worth across the distribution of households. With the aggregate real value of assets unchanged, this change is a pure redistribution, sometimes known as the "Fisher" effect. The redistribution sets each household's real net worth according to

$$q_1 = \begin{cases} \frac{m_0 + a_0(1+P_1^{\tilde{L}}\rho)}{\Pi^L} + \left(1-\delta^D\right) d_0 & \text{if} \quad a_0 < 0 \\ \frac{m_0 + \left((1-\phi_0)\frac{\Pi^L}{\Pi^H} + \phi_0(1+P_1^{\tilde{L}}\rho)\right)a_0}{\Pi^L} + \left(1-\delta^D\right) d_0 & \text{if} \quad a_0 \geq 0 \end{cases}. \tag{17}$$

For households with nominal debt $a_0 < 0$, the redistribution increases the real value of their nominal liabilities (relative to $a_0(1 + P_1^{\tilde{L}}\rho)/\Pi^H$). For household's with savings $a_0 \geq 0$, the redistribution increases the real value of their bank equity, but only for the nominal share ϕ_0 of bank assets. The surprise disinflation has no effect on the real value of the fraction $1 - \phi_0$ of the bank's assets invested in the capital stock. The expression $\left((1-\phi_0)\frac{\Pi^L}{\Pi^H} + \phi_0(1 + P_1^{\tilde{L}}\rho)\right)$ adjusts the face value of the household's claim on bank equity to reflect the gain in the real value of the bank's nominal assets. Rather than the expected real return $1+V_1-\delta$ on bank equity, the household instead earns an actual real return of $\phi_0 \frac{(1+P_1^{\tilde{L}}\rho)}{\Pi^L} + (1-\phi_0)(1+V_1-\delta)$.

Given this immediate redistribution, we consider the welfare effects along the exact equilibrium path that converges in finite time to a low inflation stationary equilibrium. The experiment is similar in spirit to Domeij and Heathcote (2004) who popularized this methodology to consider the welfare costs of a one-time change in the capital gains tax rate under imperfect insurance.

Transition path. Given an initial high inflation stationary equilibrium as described in Section 2.7 and its stationary measure ψ^H we characterize the disinflation transition equilibrium as follows. Let $\psi_1 = \psi^H$, then for $t \geq 1$, given a sequence of government expenditures $\{G_t\}$ equal to G, an inflation path $\left\{\Pi_t^L\right\}$, and real balances $\{M_t\}$ that jointly satisfy the government budget constraint (12) and aggregate demand for real balances given by (14), *a disinflation transition equilibrium* is for $t \geq 1$

1. An initial redistribution described by equation (17).

2. A sequence of measures ψ_{t+1} that satisfy (13).

3. A sequences of prices $P_t^{\tilde{L}}$, $P_t^{\tilde{e}}$, W_t, V_t, inflation $\Pi_t = \Pi_t^L$ that satisfy the no-arbitrage condition (7).

4. Decision rules $c_t(q,d,z)$, $m_t(q,d,z)$, $a_t(q,d,z)$, $e_t(q,d,z)$ and $d_t(q,d,z)$ that solve the sequence of Bellman equations (10).

5. Firms maximize profits (11) with aggregate capital demand equal to aggregate savings

$$K_t = S_t. \tag{18}$$

6. The government budget constraint (12) holds with $M_t = M_t^d$ given aggregate demand for real balances given by (14).

2.9 Model solution

For the stationary economy, we use an extended version of the endogenous grid method developed by Hintermaier and Koeniger (2010) to solve for the household decision rules under constant prices and compute the prices in which aggregate demand for nominal bonds over the implied stationary distribution of households is equal to demand from government borrowing and the capital stock.[12]

Computing the non-stationary solution for the disinflation equilibrium path is by now relatively standard. We use an approach similar to Domeij and Heathcote (2004). We look for a stationary low inflation equilibrium using the method just described. The disinflation equilibrium will converge to the low inflation equilibrium in finite time. For our calibration this is well under 150 periods,

[12]We discuss the model solution for the stationary and non-stationary economies in some detail in appendix C.

and we consider 200 period sequences of prices to characterize the full transition path. For a given sequence we can solve backwards from the low inflation equilibrium along the conjectured sequence of prices, again using the endogenous grid method to find the sequences of optimal decision rules. Then starting from the distribution of households in the initial high inflation economy, we solve the distribution forwards using the law of motion (13) and the disinflation sequences of policy rules. We look for a sequence of prices where the capital market (18) and the government budget constraint (12) clear in each period. The disinflation equilibrium solution delivers the sequence of policy rules and distributions characterizing household heterogeneity during the disinflation period.

3 Initial high inflation equilibrium

The starting point for our experiment is the high inflation period from the mid 1970s to 1981 that preceded the Volcker disinflation. We calibrate our model economy to mimic this macro environment and to match moments of household heterogeneity measured in microdata on household finances around that period.

3.1 Household finance data

To measure the pre-Volcker high inflation period, our primary source of data is the 1983 Survey of Consumer Finances (SCF) from the Federal Reserve Board. The survey consists of a representative sample of the U.S. population plus a supplemental sample of high income households drawn from a sampling frame of 5000 high-income tax payers estimated to have substantial wealth by the Internal Revenue Service's (IRS) Statistics of Income Division (SOI). The oversampling of high income households allows for a more accurate representation of the tail of the wealth distribution than comparable surveys.[13] Interviews for the 1983 SCF were conducted in person from February to August of 1983, and respondents in many cases were answering questions about their household finances in 1982. Our view is the 1983 SCF is a reasonable approximation to the wealth and income distributions in the high inflation period.[14] Although in the model the disinflation is completely credible, in practice inflation expectations even during the Volcker disinflation remained stubbornly high. So household finances in 1982 to 1983, especially portfolio positions, reflected in part the high inflation period from the late 1970s.

Using the 1983 SCF we measure components of household wealth. Participants are asked about a variety of asset and debt classes including financial assets, paper assets, liquid assets, the cash value of durable goods, consumer debt and real estate debt. We classify the debt and assets into nominal and real positions and calculate the net nominal, net real, and liquid wealth distributions.[15]

[13] See Avery et al. (1988) for a complete description of the 1983 SCF survey and methodology.

[14] Ideally we would have household finance data measured during the exact high inflation period. Unfortunately, we are not aware of any reliable household finance data covering this time period. The predecessor to the SCF was conducted in 1970 and again in 1977. In 1976 and 1977 inflation had also abated somewhat, so it is not ideal. Also, the 1983 survey design was the first to include the high income oversample needed to precisely estimate the distribution of wealth.

[15] Appendix A describes in detail our grouping of the household assets and liabilities in the 1983 SCF.

With one exception, this measurement is similar to Doepke and Schneider (2006a) for a different time period. We differ by only identifying direct nominal positions at the household level. Doepke and Schneider (2006a) use the Flow of Funds data from the Federal Reserve Board to correct for the indirect nominal positions of households, where indirect nominal wealth includes the nominal positions of the businesses on which the household has claims. They determine the indirect position using the nominal leverage ratio of the U.S. business sector which they define as the nominal debt position per dollar of equity. This correction is well suited to their goal of characterizing the nominal position of the household sector and cohorts of the household sector, but will be substantially less accurate for characterizing the distribution of the nominal wealth among households. We believe that the bias created by not correcting for indirect nominal positions will be small. In the 1983 SCF only 34.9% of households have any claims to public or private equity, and of those, the median equity share of net worth was only 16.6%.

3.2 Calibration of high-inflation equilibrium

Period length, duration and high inflation. The period length is one year. To capture the longer duration of household debt contracts we use bonds with a constant decay rate governed by ρ. If ρ is equal to 0 then the bonds are equivalent to the one period bond that is typical in this literature. In our baseline calibration we set $\rho = .89$, which implies a duration of 4.5 years to match the average duration of household nominal liabilities in the U.S. for this period as documented by Doepke and Schneider (2006a). For the initial high inflation equilibrium we consider a gross inflation rate target of $\Pi^* = 1.10$. This is roughly in line with price inflation at the beginning of Volcker's term as Chairman of the Federal Reserve Board.

Preferences. We specify household preferences with relative risk aversion σ over a CES aggregate of consumption, real balances and durables so that

$$u\left(c_t, m_t, d_{t-1}\right) \equiv \frac{1}{1-\sigma}\left(\left(\omega c_t^{\frac{\eta-1}{\eta}} + (1-\omega)\, m_t^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}\theta} d_{t-1}^{1-\theta}\right)^{(1-\sigma)} \tag{19}$$

With these preferences the elasticity of substitution between consumption and real balances η will turn out to be the interest elasticity of money demand, since unconstrained households would choose

$$m = \left(\frac{1+i_t}{i_t}\frac{1-\omega}{\omega}\right)^{\eta} c\ . \tag{20}$$

Lucas (2000) finds $\eta = 0.5$ to be a reasonable approximation for the aggregate interest elasticity of demand for M1, and he uses this value when computing the welfare costs of inflation. Other estimates put the elasticity closer or equal to 1.[16] We choose $\eta = 0.5$ and examine the sensitivity of our results to alternative elasticities. The parameter $\omega \in [0, 1]$ scales the liquidity value of real balances with $\omega = 1$ implying no liquidity value of money and thus zero demand for real balances.

[16] See Hoffman et al. (1995), Holman (1998), Lucas (2000) and citations therein.

With η fixed, we set $\omega = 0.988$ to target the ratio of real balances to output in the high inflation stationary distribution. We choose the discount factor $\beta = 0.9391$ to generate a steady state capital to output ratio of approximately 3.3 in equilibrium. In line with the literature, we set the coefficient on relative risk aversion, σ, to be 2. Following, Fernández-Villaverde and Krueger (2011) we set $\theta = .81$ to target a share of the household budget spent on non-durables of 20%.

Durable goods. We follow Hintermaier and Koeniger (2010) and set quadratic adjustment costs for durables:

$$\Psi\left(d_t, d_{t-1}\right) = \frac{\kappa}{2}\left(\frac{d_t - (1-\delta_d)d_{t-1}}{d_{t-1}}\right)^2 d_{t-1} \tag{21}$$

The parameter κ represents the cost of adjusting durable holdings. Like Hintermaier and Koeniger (2010), we set κ to .05 to represent the typical transaction costs for buying or selling a home. The depreciation rate on durable goods we set to .03 in line with standard values from the literature (Sterk and Tenreyro (2018) set durable depreciation to .04 while Hintermaier and Koeniger (2010) set it to .02). We set the securitization rate on durable goods, μ, to be .7 to capture the securitization constraints facing the average consumer. Luengo-Prado (2006) finds that in the 1970s, the average down payment for U.S. households was approximately 26%.

Production. For production we use a Cobb-Douglas production function

$$F\left(K, N\right) = K^{\alpha} N^{1-\alpha}$$

with capital share $\alpha = 0.33$, which is roughly in line with long run average of capital income to output. We choose an annual depreciation rate of $\delta^K = 0.06$ to generate an investment to output ratio of roughly 0.2 given the capital to output ratio.

Remaining parameters. We set τ to be .20 to target a government spending to GDP ratio of approximately .2 In the model with money we force government spending, G, to adjust with the change in seignorage revenue as discussed in Section 2.5. In practice, the change in seignorage revenue results in small change in government spending as a share of output since most government spending is financed by tax revenue.

The only parameters remaining are the real borrowing limit and the parameters of the Markov chain governing idiosyncratic labor efficiency, which we calibrate to approximately match the distribution of net worth. We follow Domeij and Heathcote (2004) and choose a 3 state Markov chain with a relatively high productivity state with less persistence. Section B gives the details on the calibration of the Markov process. We summarize all of the calibration parameters in Table 1.

3.3 Model fit in the high-inflation equilibrium

In Table 2 we compare the wealth distribution of households in the 1983 SCF with our calibrated high inflation economy. Instead of expressing the distribution in dollars, we instead describe points

Parameter		Value
A. Preferences		
Discount factor	β	0.94
Risk aversion	σ	2.00
Money demand elasticity	η	0.50
Weight on real balances	ω	0.99
Nondurable share	θ	0.81
B. Durables		
Adjustment cost	κ	0.05
Depreciation	δ^D	0.03
C. Production		
Capital share	α	.33
Depreciation	δ^K	0.06
D. Other parameters		
Income risk	z_i, P	Domeij and Heathcote (2004)
Bond decay rate	ρ	0.89
Secured borrowing	μ	0.70
Taxes	τ	.20

Table 1: Calibration parameters

along the Lorenz curve. For example, in the 1983 SCF, the top 10 percent of households ordered by their net worth, owned 66.7 percent of total net worth, and the bottom 50 percent of households owned only 3.8 percent of total net worth. We hold the ordering by net worth fixed across all of the variables. We also report the Gini statistic for each of our variables. The Gini measures the area between a 45 degree line and the Lorenz curve for each variable. When all shares are non negative, it will be between zero and one, with zero indicating perfect equality and one indicating perfect inequality. In our case some households will be net borrowers so the Gini coefficient could in principle be above one.

In 1983, as in other years, the distribution of net worth is skewed, with the top one percent of households owning 31 percent of total net worth. The precautionary savings motive in our model is able to replicate the inequality in the wealth distribution, but fails to generate the substantial wealth accumulation in the very far tail. This is typical in this class of model where precautionary motives alone cannot account for the extreme wealth accumulation of the very rich. Models with an additional savings motives from a rare and transient superstar state, entrepreneurship with financial constraints, or bequest motives are better able to replicate this behavior. Our model misses some of the debt accumulation for the poorest 10% of the households whose share of net worth in the data is negative.

In the data, net nominal positions are negative for half of the population versus 48 percent in the high inflation steady state in our model. This reflects secured borrowing in the form of mortgages,

	Lowest, ordered by net worth			Highest, ordered by net worth			
Percent of total	10%	25%	50%	10%	5%	1%	Gini
A. 1983 SCF							
Net worth	-0.14	0.12	3.82	66.65	54.56	31.22	0.78
Liquid assets	0.31	1.07	6.58	50.18	34.80	13.97	0.81
Nominal wealth	-4.83	-9.22	-34.8	121.40	95.44	43.06	—
Real wealth	0.15	0.70	5.71	65.11	53.93	31.97	0.76
B. Baseline Calibration							
Net worth	0.58	5.55	8.83	47.75	28.90	7.25	0.651
Liquid assets	2.50	16.56	25.53	30.92	18.12	4.39	0.40
Nominal wealth	-10.12	-12.07	-7.17	50.25	29.93	7.38	—
Real wealth	0.92	6.10	9.33	47.68	28.87	7.24	0.64

Note: 1983 Survey of Consumer Finances. Adjusted net worth is total net worth less the value of any durable assets or secured borrowing against these assets. See appendix for detailed descriptions. We omit the Gini coefficient for nominal net worth because it is negative for almost 75 percent of households.

Table 2: Distribution of Wealth: Data and Model

which is captured in our model by secured borrowing against durables. This is important when thinking about welfare since households with nominal debt contracts stand to lose in a sudden disinflation. In a model that incorporates secured nominal borrowing against durable real assets, this means welfare losses occur across the income distribution, not just among the poor.

4 The welfare costs of the Volcker disinflation

In November of 1980 to very early 1981 the Volcker disinflation began in earnest. Lasting roughly two years, inflation declined from over 10 percent to a little less than 4 percent.[17] We consider this disinflation policy in the model as an unexpected and immediate shift in the monetary policy stance. The government announces a 3 percent inflation target and commits to whatever it takes to achieve this new lower inflation rate, i.e., at the beginning of period $t = 1$, $\Pi^* = 1.03 < \Pi^H$. To implement this policy the government accommodates the change in demand for real balances induced by the new inflation target and chooses government spending G_t to balance the budget. We quantify the effects ofthe Volcker disinflation by computing the transition equilibrium path defined in section 2.8 to the new low inflation long-run equilibrium.

All things equal, everyone benefits from a lower inflation tax, but, the first period of the transition imposes a one time wealth redistribution. Those with net nominal liabilities find the real burden of their liabilities unexpectedly higher. Those with net nominal savings receive an unexpected windfall. Borrowers and savers are also affected differently by the slight increase in the real interest rate stemming from the non-neutrality of money. For each household, the welfare

[17] Goodfriend and King (2005) recount the course of events and policy commitments leading up to and through the "incredible Volcker disinflation." See also Lindsey et al. (2005)

	High inflation	Low inflation
	A. Interest rates	
i	14.87	7.59
r	4.42	4.45
	B. Aggregates	
Y^*	1	1.00*
C/Y	0.70	0.71
K/Y	3.66	3.65
Y/M	4.66	3.39
D/Y	2.26	2.27
G/Y	0.17	0.16

Note: * rounded from 0.999

Table 3: High Inflation and Low Inflation Steady State Comparison

costs of the disinflation will depend on whether the benefit of the lower inflation tax is enough to compensate them for the burden of the redistribution and the change in borrowing costs.

We compute a simple conditional welfare measure that asks on the eve of the inflation reform what wealth equivalent each household would require to be indifferent between the economy with the disinflation and a counterfactual economy which remains in the high inflation equilibrium permanently. We also consider long run measures that look at the welfare consequences if households could skip the transition period and jump right to the low inflation equilibrium.

4.1 Long-run low inflation equilibrium

Before we measure the disinflation period in the model, we see where it will eventually lead the economy. In Table 3 we compare the high inflation and low inflation long-run economies that represent the beginning and ultimate ending of the disinflation. For comparison, output is normalized to 1 in the high inflation economy. The first thing we notice is that in the model with money, monetary policy is not super-neutral. In the long run, the reduction in inflation slightly raises the real rate and thus lowers the capital stock and by extension output. This occurs because as the inflation tax decreases, households shift their portfolios away from other assets and towards money (aggregate real balances increase approximately 38% in the low inflation steady state). This portfolio rebalancing puts upwards pressure on the real interest, a version of the Tobin (1965) effect.[18] Later, we will shut down this portfolio rebalancing channel by considering a cashless limit economy with $\omega = 1$ where the liquidity value of money is zero.

[18]See Algan and Ragot (2010) who discuss this property in more detail.

4.2 Short-run conditional welfare

The unanticipated disinflation begins in period $t = 1$. We define $V_1^L(q_1, d_0, z_1)$ as the value of a household in period $t = 1$ with real resources real resources q_1 already reflecting redistribution from the unexpected change in Π_1 according to equation (17). The value V_1^L can be written as a function of sequences of consumption, real money balances and durables determined along the disinflation equilibrium path

$$V_1^L(q_1, d_0, z_1) = E\left[\sum_{k=0}^{\infty} \beta^k u\left(c_{t+k}^L, m_{t+k}^L, d_{t+k-1}^L\right) | t = 1, z_1\right].$$

Relative to what was anticipated under high inflation, q_1 reflects a one-time windfall gain to those with positive net worth and a loss to borrowers. Next, we define for the same household the counterfactual value $V_1^H\left(q_1', d_0, z_1\right)$ of remaining in the high inflation environment forever. Here d_0 and z_1 are identical, but q_1' is the real net worth with the high inflation that was expected. This value V_1^H can be written as a function of sequences of consumption, real money balances and durables in the economy where inflation remains high:

$$V_1^H\left(q_1', d_0, e_1\right) = E\left[\sum_{k=0}^{\infty} \beta^k u\left(c_{t+k}^H, m_{t+k}^H, d_{t+k-1}^H\right) | t = 1, z_1\right].$$

Because the initial high inflation environment is stationary $V_0^H = V_1^H$.

We define implicitly the wealth equivalent conditional welfare change $\Delta_q(q_1, d_0, z)$ as the adjustment to their initial real wealth needed to make the counterfactual environment equivalent to a transition to a new low inflation steady state

$$V_1^H\left((1 + \Delta_q) q_1', d_0, e_1\right) = V_1^L(q_1, d_0, z_1)\ . \tag{22}$$

When $\Delta_q < 0$, this implies that they would be willing to sacrifice fraction Δ_q of their wealth in order to avoid going through the transition while $\Delta_q > 0$ means they need to be compensated with an additional fraction Δ_q of their resources to stay in the high inflation steady state. We plot in Figure 1 the wealth equivalent for each household versus the initial nominal wealth position of the household from a simulation of the model with 5000 households.[19]

Despite the disinflation raising output and consumption in the medium and long run, the gains are not spread equally across the distribution. It is apparent from Figure 1 that nominal borrowers bear the cost from the one time redistribution from the surprise disinflation with low income nominal borrowers bearing the worst costs—up to 120% of their wealth for the poorest households in the model without money.

In Table 4 we tally who would vote for the Volcker disinflation? Since 48 percent of households

[19] Note that these households also differ in their initial durables position which also affects the welfare consequences of the disinflation. Thus, two households with similar initial nominal wealth positions may have very different welfare costs.

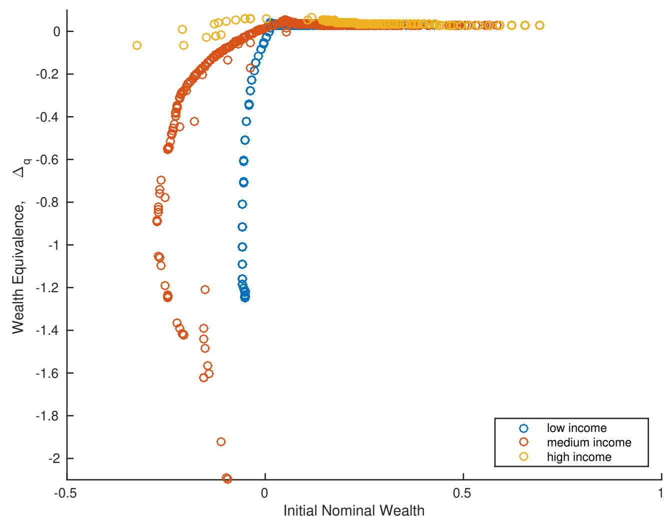

Figure 1: Permanent wealth equivalent of Volcker disinflation as percent

	Percent that prefer high inflation		Percent borrowers	Percent of population
	Short run	Long run		
Total economy	14.00	4.39	48.17	100
Low income	43.67	32.54	44.77	11.69
Middle income	11.14	0.76	55.21	76.92
High income	2.85	0.00	4.12	11.38

Table 4: Preference for Disinflation Policy

are borrowers, most stand to lose from the redistribution. However, for most, the lower inflation tax compensates them for their loss. Only 29 percent of the borrower households, at least according to these preferences, would prefer to remain in the high inflation equilibrium than start a disinflation period. These voters are concentrated among the low and middle earners, specifically among the segment of those groups that are borrowers or have little net worth.

4.3 Long-run measures

We then ask whether households would choose differently if they could immediately reach the low inflation equilibrium. To do this we examine the same individuals in the counterfactual and disinflation economies long after the disinflation equilibrium has reached its permanent state. In Table 4 we report the same vote far into the future, and see that about 95 percent of households now prefer the low inflation equilibrium in the model with money. We note that our experiment *does* include the one-time initial redistribution, but many years have passed and households have sufficient time to readjust their savings. Most prefer the low inflation economy given the lower cost of liquidity from the reduction in the inflation tax. What may be surprising is the 4.39 percent of households that would still prefer to remain in the high inflation economy. These are only the poorest households who suffer from the slight rise in the real interest rate; had the interest rate been unaffected, these households would also prefer low inflation in the long run.

5 Sensitivity of welfare to alternative calibrations

The baseline welfare results depend on the trade-off between the benefit from lowering the inflation tax, the cost of the redistribution and the increase in the real interest rate. In this section, we explore the sensitivity of our results to changing key parameters that adjust the size of the redistribution and the size of the inflation tax. We also apply the quantitative framework to consider the distribution of welfare effects of an *increase* in the inflation target.

5.1 Parameters

The more money held by households, the greater the burden of the inflation tax. To generate demand for money, we include it directly in the household's utility function. When unconstrained, a household's money demand is proportional to their consumption (as shown in equation 20). We interpret the demand for money as demand for liquidity. Households need money in order to consume and pay bills. The amount of money held will be determined by ω. When $\omega = 1$, households hold no money and are unaffected by the inflation tax. We compare our baseline welfare results from section 4 to the same calibration with no liquidity value of money, $\omega = 1$. In this calibration, the results are solely driven by the effect of the redistribution. Households get no benefit from the lower inflation tax.

The duration of the debt contracts will determine the size of the redistribution. In the model, the duration is governed by ρ. When $\rho = 0$ households borrow in one period nominal bonds. The effect of increasing ρ can be seen in the law of motion for wealth, equation (9). When $\rho = 0$ and there is an unexpected disinflation, households are only affected by the change in Π_t. When $\rho > 0$ they are also affected by the unexpected change in the price of nominal bonds $P_t^{\tilde{L}}$. As nominal interest rates fall during the disinflation, the price of nominal bonds, $P_t^{\tilde{L}}$, will increase. For borrowers, the increase in $P_t^{\tilde{L}}$ implies an increase in their real debt burden. Below we present the welfare results from a calibration that uses 1 period nominal bonds instead of the long duration asset. We compare the welfare results to our baseline calibration in which the duration was set to 4.5 years to match the average duration of nominal assets in the U.S. as documented by Doepke and Schneider (2006a). With one period nominal bonds, the size of the redistribution will be smaller and households are more likely to be compensated by the benefit of the lower inflation tax.

5.2 Welfare results

For each economy, Table 5 presents the results from a straight up or down vote of whether households prefer to go through the disinflation or remain in the high inflation steady state. There is a stark difference between our baseline calibration and the economy with no money. Without

	Percent that prefer high inflation			Percent of borrowers that prefer high inflation		
	baseline	cashless limit	1-period bonds	baseline	cashless limit	1-period bonds
Total economy	14.00	58.48	6.18	29.07	100.00	12.83
Low income	43.67	53.64	40.23	97.56	100.00	89.87
Middle income	11.14	66.87	1.92	20.18	100.00	3.48
High income	2.85	6.82	0.00	69.22	100.00	0.00

Table 5: Alternative Experiments: Preference for Disinflation Policy

	p5	p10	p25	p50	p75	p90	p95	mean
Baseline	36.15	4.86	-2.10	-2.74	-3.24	-4.03	-4.43	2.64
Cashless limit	49.72	8.61	0.61	0.15	-0.23	-98.58	-99.73	-3.40
1-period bonds	3.87	-2.14	-2.51	-3.46	-4.24	-4.35	-4.43	-2.27

Table 6: Alternative Experiments: Distribution of welfare costs

money, there is no decrease in the inflation tax and there is no change in the real interest rate. The only effect of the disinflation is to redistribute wealth away from borrowers towards saver households. Without the lower inflation tax there is nothing to compensate borrowers for their loss in the redistribution and, as a result, 58.48 percent of households would prefer to remain in the high inflation steady state versus 14.00 percent in our baseline calibration. Because middle income households actually have a higher share of borrowers (55.21 percent) than low income households (44.77 percent), more middle income households vote for the high inflation economy than in the baseline calibration where low income households were more likely to vote for high inflation. In the economy with one period bonds, instead of the 4.5 year duration in the baseline economy, the size of the redistribution induced by the disinflation is smaller. As a result, the decline in the inflation tax is more likely to compensate borrower households for their losses. Consistent with this, we see the share of households that vote for the transition fall. However, the middle income and high income households see a larger decrease in the share who vote for high inflation. Amongst low income households, a substantial amount, 40.23 percent, still prefer to remain in the high inflation steady state. Some of this is driven by the increase in the steady state real interest rate which impacts low income households the most. Even if low income households could skip the transition period and jump straight to the low inflation steady state, 32.54 percent of them would still prefer to remain in the high inflation steady state.

Turning our attention to the distribution of the welfare costs, Table 6 presents moments from the distribution of wealth equivalents, Δ_q, as described in section 4.2. In our baseline calibration, Δ_q is negative for more than 75% of the population indicating that most households would be willing to sacrifice some of their wealth to go through the disinflation. At the 95th percentile, households would sacrifice 4.43 percent of their wealth in exchange for the transition to low inflation. However, the costs are much higher at the other end of the distribution. At the 5th percentile households need to be compensated 36.15 percent of their wealth to be indifferent between the transition and staying in the high inflation steady state. On average, households need to be compensated 2.64 percent in the baseline calibration.

In the cashless economy, the magnitudes of the wealth equivalents are much larger at the tails of the distribution. Some households would be willing to sacrifice almost all of their wealth to go through the redistribution. This is not surprising if you consider that some households stand to receive a substantial windfall with no negative effects from a higher real interest rate. On the flip side, some households need to be compensated a substantial amount to go through the redistribution, 49.72 percent of their wealth for the household at the 5th percentile of wealth equivalents. Another difference with the baseline economy, is that households in the middle of the distribution, between the 25th and 75th percentile, are roughly indifferent between staying in the high inflation steady state and going through the redistribution. These households do not stand to gain or lose much from the redistribution. In the baseline economy, they strongly prefer the transition because they stand to gain from the lower inflation tax, but in the absence of money they are indifferent.

In the economy with one period bonds, the distribution of wealth equivalents look similar to the baseline calibration except in the left tail, below the tenth percentile. These are households who lost a lot from the redistribution in the baseline calibration and needed to be compensated substantially (36.15 percent for the fifth percentile). However, with one period bonds the size of the redistribution is smaller so borrower households have less to lose and stand to benefit from the lower inflation tax. Now the fifth percentile of households only needs to be compensated 3.87 percent to be indifferent between the redistribution and staying in the high inflation steady state. With a smaller redistribution, the average wealth equivalence switches sign. While households needed to be compensated an average of 2.64 percent under the baseline calibration, with 1 period bonds they would be willing to give up an average of 2.27 percent of their wealth. The differences between the baseline calibration and the alternative experiments highlight the importance of capturing all three channels (the redistribution, the decrease in the inflation tax and the increase in the real interest rate) when considering the welfare costs of a disinflation.

5.3 Raising the inflation target

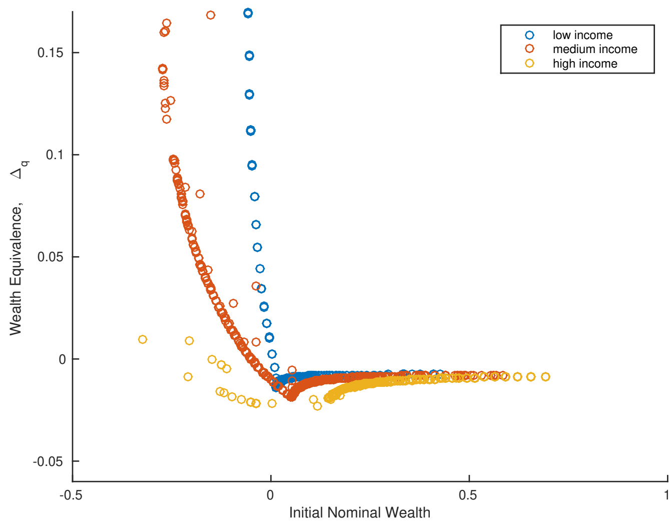

Figure 2: An Inflationary period from 10 to 13 percent Inflation

Briefly, we use our model to consider one final experiment. In light of recent concerns about low inflation leaving monetary policy close to the zero lower bound on interest rates, we consider an increase in the inflation rate from 10 to 13 percent.[20] An inflationary period has also been considered as a way to decrease the real debt burden of borrowers and stimulate aggregate demand. Table 7 shows the results from a straight up vote on whether or not to increase the inflation target. Almost ninety percent of households prefer to remain at low inflation rather than go through the inflation period. As in our baseline experiment, the welfare costs are distributed unequally across the distribution, 43 percent of poor households would prefer to go through the inflation as the benefits from decreasing their real debt burden offset the increase in the inflation tax. Figure 2 presents a scatter plot of the short-run wealth equivalent measure from equation (22) using a simulation of the inflationary experiment. Low income households, represented by the blue dots, gain the most from the inflation, up to 15 percent of their wealth. Importantly, even though the poorest households with a high marginal propensity to consume gain from the redistribution, the increase in the inflation tax increases the price of consumption goods and makes aggregate consumption decline. The effects on output however are positive as households rebalance their portfolios away from money and towards assets causing the real interest rate to decline and the capital stock to increase.

[20] We consider a change from 10 to 13 instead of from 2 to 5 because our initial steady state is calibrated to match the economy in the late 70s when inflation was often in excess of 10 percent. We also consider an experiment in which we change the steady state rate of inflation from 2 to 5 and find very similar welfare results.

	Percent that prefer low inflation		Percent borrowers	Percent of population
	Short run	Long run		
Total economy	87.74	96.67	48.17	100
Low income	57.42	75.39	44.77	11.69
Middle income	90.79	99.42	55.21	76.92
High income	98.25	100.00	4.12	11.38

Table 7: Vote on increasing the inflation target

6 Conclusion

We examine the welfare effects of a permanent and unexpected change in the inflation target of the central bank. An unexpected decline in the inflation rate redistributes wealth from nominal borrowers towards savers, but it also lowers the burden of the inflation tax for all households. We focus on the welfare effects stemming from the resulting redistribution of wealth, the change in the inflation tax and equilibrium effects on the real interest rate in a benchmark model with flexible prices and a credible central bank.

We build an Aiyagari (1994) model modified with a consumer portfolio choice between a long term nominal bond, real durable goods, and money which we use to analyze the welfare effects of an unexpected change in the steady state inflation rate. Borrowing must be secured against real durable goods meaning that even wealthy households can have negative nominal wealth positions, as is often the case for US households with a nominal, fixed interest mortgage secured against their home. The change in the inflation rate redistributes wealth from households with net nominal liabilities towards those with positive net nominal assets.

We use the model to consider a disinflation from 10 to 3 percent, roughly the disinflation experienced during the "Great Volcker Disinflation" in the early 80s. We compare the welfare results from several calibrations. In the first, we calibrate the duration of nominal debt and the liquidity value of money to match characteristics of the US wealth distribution in the early 1980s. The disinflation is a redistribution of wealth from borrowers to savers, but most households are compensated by the lower inflation tax. Weighing the two effects only 14 percent of households prefer to remain in the high inflation steady state, though these are concentrated among low income households, 44 percent of whom prefer the high inflation steady state to the transition

We then calibrate two alternative versions of our baseline model: a cashless limit in which there is no benefit from a lower inflation tax and the disinflation is a pure wealth redistribution, and a version with only short term nominal debt in which the size of the redistribution is smaller. With a 1 period bond and a smaller distribution only 6.18 percent of households prefer to stay in the high inflation steady state, less than half that of the baseline calibration. The cashless version of the economy also produces substantially different results. Without the lower inflation tax, there is nothing to compensate borrowers for the their losses in the redistribution and 58 percent of households prefer to remain in the high inflation steady state. These results suggests that it is crucial to capture the duration of assets and the change in the inflation tax when considering the welfare consequences of changing the inflation rate.

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A Data appendix

Table A.1 reports the real and nominal categorization of assets and liabilities measured in the SCF.

Assets	Liabilities
A. Nominal assets and liabilities	
Liquid:	**Secured borrowing:**
Cash in checking accounts	Home mortgages
Cash in savings or share accounts	Amount owed against land contract notes
Money market and call accounts	
IRA or Keogh accounts	Amount outstanding on other property mortgages
Certificates of Desposit	
U.S. Savings Bonds	
Non liquid:	**Unsecured borrowing:**
Face value of bonds	Amount outstanding on loans other than mortgages[1]
Loans owed to household and gas leases	Credit card debt
Aggregate gross value of land contracts and notes	Amount owed on lines of credit
Thrift type pension account assets	
B. Real assets	
Durables:	
Home	
Other properties	
Vehicles	
Financial:	
Stocks and mutual funds	
Trust accounts	
Business:	
Net value of business with management interests	

Table A.1: Categorization of SCF household assets and liabilities

Note: classification of assets and liabilities variables in 1983 Federal Reserve Board Survey of Consumer Finances.
[1]Also subtracts loans against life insurance policies.

B Calibration of Markov Process

The calibration of the Markov process follows Domeij and Heathcote (2004). Equation (23) gives the income states for our productivity process. There are three income groups with the income of the highest group normalized to 1.

$$z \in [0.0381; 0.2155; 1] \tag{23}$$

Equation (24) gives the transition matrix describing how households transition between states. An element of the matrix π_{ij} describes the probability that the household transitions between from state i to state j.

$$\begin{array}{cccc} & z_1 & z_2 & z_3 \\ z_1 & .975 & .025 & 0 \\ z_2 & .0038 & .9925 & .0037 \\ z_3 & 0 & .025 & .975 \end{array} \tag{24}$$

Given the transition matrix, in steady state, 11.69 of the population will be low income , 76.92 will be middle income, and 11.38 high income. As described in table 2, the process does a good job of replicating the distribution of income and wealth across households.

C Solution Algorithm

C.1 Model Solution

C.1.1 Household Decision Rules

Let $V_t(q_t, d_{t-1}, z_t)$ be the value function of an individual with real net worth q_t, existing durables d_{t-1}, and labor efficiency z_t. Then V_t satisfies the Bellman equation given by equation 10 subject to the individual's budget constraint (8) and real borrowing constraint (5). In addition, we enforce and Inada conditions give non negativity constraints

$$m_t \geq m_0 \geq 0 \quad \text{and} \quad d_t \geq d_0 \geq 0 \tag{25}$$

The law of motion for resources is given by equation (9).

To simplify notation we define the discounted expected continuation value

$$f_t\left(q', d_t, z_t\right) \equiv \beta E\left[V_{t+1}\left(q', d_t, z_{t+1}\right) | z_t, t\right] \ . \tag{26}$$

Let ν_t be the multiplier on the secured borrowing constraint, and let κ_t and χ_t be the multipliers on the non negativity constraints for money and durables respectively. Then the KKT conditions (omitting the budget constraint and multiplier λ_t) are

$$\begin{aligned} \frac{f_{qt}+\nu_t}{\Pi_{t+1}} &= \frac{u_{ct}}{1+i_t} && (27) \\ u_{mt} + \frac{f_{qt}}{\Pi_{t+1}} + \kappa_t &= u_{ct} && (28) \\ f_{qt}(1-\delta_D) + f_{dt} + \nu_t \mu (1-\delta_D) + \chi_t &= u_{ct}(1+\Psi_{1t}) && (29) \\ \nu_t\left(\frac{a_t}{\Pi_{t+1}} + b + \mu(1-\delta_D) d_t\right) &= 0 \qquad \nu_t \geq 0 \\ \kappa_t(m_t - m_0) &= 0 \qquad \kappa_t \geq 0 \\ \chi_t(d_t - d_0) &= 0 \qquad \chi_t \geq 0\ . \end{aligned}$$

Combining the Euler equations for bonds (27) and durables (29) we can write an equation for the multiplier ν_t given q_{t+1}, d_t, z_t and possibly χ_t

$$\begin{aligned} f_{qt}(1-\delta_D) + f_{dt} + \nu_t\mu(1-\delta_D) + \chi_t &= (1+i_t)\frac{f_{qt}+\nu_t}{\Pi_{t+1}}(1+\Psi_{1t}) \\ \nu_t\left(\frac{1+i_t}{\Pi_{t+1}}(1+\Psi_{1t}) - \mu(1-\delta_D)\right) - \chi_t &= -f_{qt}\left(\frac{1+i_t}{\Pi_{t+1}}(1+\Psi_{1t}) - (1-\delta_D)\right) + f_{dt}\ . && (30) \end{aligned}$$

We can determine f_{qt} and f_{dt} from envelope conditions

$$f_{qt}\left(q_{t+1},d_t,z_t\right) = \beta E_t\left[u_{ct+1}\right] \tag{31}$$
$$f_{dt}\left(q_{t+1},d_t z_t\right) = \beta E_t\left[u_{dt+1}-u_{ct+1}\Psi_{2t+1}\left(d_{t+1},d_t\right)\right] \tag{32}$$

CES Preferences We use a CES specification for utility given in equation 19. For notation we define

$$\Upsilon_t \equiv \omega c_t^{\frac{\eta-1}{\eta}}+(1-\omega)\, m_t^{\frac{\eta-1}{\eta}}$$

so

$$\begin{aligned}
u_{ct} &\equiv \Upsilon_t^{\frac{1-\eta(1-\theta)-\sigma\eta\theta}{\eta-1}} d_{t-1}^{(1-\sigma)(1-\theta)}\theta\omega c_t^{\frac{-1}{\eta}}\\
u_{mt} &\equiv \Upsilon_t^{\frac{1-\eta(1-\theta)-\sigma\eta\theta}{\eta-1}} d_{t-1}^{(1-\sigma)(1-\theta)}\theta(1-\omega)\, m_t^{\frac{-1}{\eta}}\\
u_{dt} &\equiv \Upsilon_t^{\frac{\eta\theta-\eta\theta\sigma}{\eta-1}} d_{t-1}^{-\theta-\sigma(1-\theta)}(1-\theta)
\end{aligned}$$

Now we can write equations (27) and (28) as

$$\frac{f_{qt}+\nu_t}{\Pi_{t+1}} = \frac{\Upsilon_t^{\frac{1-\eta(1-\theta)-\sigma\eta\theta}{\eta-1}} d_{t-1}^{(1-\sigma)(1-\theta)}\theta\omega c_t^{\frac{-1}{\eta}}}{1+i_t} \tag{33}$$
$$\frac{f_{qt}}{\Pi_{t+1}}+\kappa_t = \Upsilon_t^{\frac{1-\eta(1-\theta)-\sigma\eta\theta}{\eta-1}} d_{t-1}^{(1-\sigma)(1-\theta)}\theta\left(\omega c_t^{\frac{-1}{\eta}}-(1-\omega)\, m_t^{\frac{-1}{\eta}}\right) \tag{34}$$

Adjustment Costs We use the a quadratic specification for adjustment costs given in equation (21) as in Hintermaier and Koeniger (2010) and

$$\Psi_1\left(d_t,d_{t-1}\right)=\rho\left(\frac{d_t}{d_{t-1}}-(1-\delta_d)\right)$$

and

$$\Psi_2\left(d_t,d_{t-1}\right)=\frac{\rho}{2}\left((1-\delta_d)^2-\left(\frac{d_t}{d_{t-1}}\right)^2\right)$$

so that with $\nu_t=0$ then

$$\frac{1+i_t}{\Pi_{t+1}}\frac{\rho}{d_{t-1}}=\frac{-f_{qt}\left(\frac{1+i_t}{\Pi_{t+1}}\left(1-\rho\left(1-\delta_D\right)\right)-\left(1-\delta_D\right)\right)+f_{dt}}{f_{qt}d_t}$$

with $\nu_t \geq 0$ then

$$\frac{1+i_t}{\Pi_{t+1}}\frac{\rho}{d_{t-1}}=\frac{f_{dt}-\frac{1+i_t}{\Pi_{t+1}}\left(1-\rho\left(1-\delta_D\right)\right)\left(f_{qt}+\nu_t\right)+\left(1-\delta_D\right)\left(f_{qt}+\mu\nu_t\right)}{d_t\left(\nu_t+f_{qt}\right)}$$

C.1.2 Interior Solution

When all constraints are slack use (30) to show

$$f_{dt} = f_{qt}\left(\frac{1+i_t}{\Pi_{t+1}}\left(1+\Psi_{1t}\right)-\left(1-\delta_D\right)\right)$$

so that given q_{t+1}, z_t, d_{t-1} and functions $f_{qt}\left(q_{t+1}, d_t, z_t\right)$ and $f_{dt}(q_{t+1}, d_t, z_t)$ one can solve for d_t.

Also with non negativity constraints on m_t and d_t slack using (33) and (34) we have

$$\frac{m_t}{c_t} = \left(\frac{1+i_t}{i_t}\frac{1-\omega}{\omega}\right)^{\eta} . \tag{35}$$

We can substitute back into (33) and solve for consumption in terms of $q_{t+1}, z_t, d_t, d_{t-1}$

$$c_t = \left(\frac{\left(\omega + (1-\omega)\left(\frac{1+i_t}{i_t}\frac{1-\omega}{\omega}\right)^{\eta-1}\right)^{\frac{1-\eta+\eta\theta-\sigma\eta\theta}{\eta-1}} d_{t-1}^{(1-\theta)(1-\sigma)}\theta\omega}{\frac{1+i_t}{\Pi_{t+1}} f_{qt}}\right)^{\frac{1}{1-\theta(1-\sigma)}} . \tag{36}$$

Money demand (35) will be proportional to consumption (36). Given q_{t+1} and now m_t and d_t we can solve for a_t using (9) as

$$a_t \;=\; \Pi_{t+1}\left(q_{t+1} - (1-\delta_D)\, d_t\right) - m_t \tag{37}$$

and given q_{t+1}, c_t, m_t, a_t, d_t and d_{t-1} using the budget constraint (8)

$$q_t = c_t + \frac{a_t}{1+i_t} + d_t + \Psi\left(d_t, d_{t-1}\right) + m_t - w_t z_t\left(1-\tau_t\right) - T_t . \tag{38}$$

C.1.3 Binding Collateral Constraint

Note that given q_{t+1} from the collateral and non negativity constraints we know d_t must lie in the interval

$$d_t \in \left[d_0, \frac{q_{t+1} - \frac{m_t}{\Pi_{t+1}}}{(1-\delta_D)(1-\mu)}\right] .$$

If the interior solution for d_t is above the upper bound we know that the RHS of (30) is positive over the interval and the collateral constraint binds $\nu_t > 0$.[21] In this event, we look for a solution where the collateral constraint is binding and both non negativity constraints are slack.

Slack non negativity conditions We first look for a solution where $\kappa_t = \chi_t = 0$. If the economy were cashless we could determine d_t and thus ν_t directly from the borrowing constraint. In the monetary economy d_t and m_t are jointly determined with the borrowing constraint placing a restriction on their relationship. Suppose we knew $\nu_t > 0$, then from (30) we can determine d_t and thus f_q . From (33) with $\nu_t > 0$, and (28) with $\kappa_t = 0$ we have

$$\frac{m_t}{c_t} = \left(\frac{\omega}{1-\omega}\left(1 - \frac{1}{1+i_t}\frac{f_{qt}}{f_{qt}+\nu_t}\right)\right)^{-\eta}$$

[21] Since $\chi_t \geq 0$ and $\nu_t > 0$ when the collateral constraint binds, this requires of course that $\mu < \frac{1+i_t}{\Pi_{t+1}}\frac{1+\Psi_{1t}}{1-\delta_D}$, i.e. that the secured borrowing constraint cannot be too loose. An Inada condition on d and our choice of d_0 sufficiently small will ensure that $d_t \geq d_0$. If there were some autonomous level of durable consumption, as in Hintermaier and Koeniger (2010), sufficiently large, we would need to verify that d_t does not violate its lower bound with the collateral constraint slack.

and substituting back into the first order condition we an equation for m_t in terms of d_t and ν_t

$$\begin{aligned} m &= \left(\frac{\left(\omega\Omega_t^{\eta-1} + (1-\omega)\right)^{\frac{1-\eta(1-\theta)-\sigma\eta\theta}{\eta-1}} d_{t-1}^{(1-\sigma)(1-\theta)}\theta\omega}{\frac{1+i_t}{\Pi_{t+1}}\left(f_{qt}+\nu_t\right)\Omega_t} \right)^{\frac{1}{(1-\theta)+\theta\sigma}} \\ &= \left(\frac{\left(\omega\Omega_t^{\eta-1} + (1-\omega)\right)^{\frac{1-\eta(1-\theta)-\sigma\eta\theta}{\eta-1}} d_{t-1}^{(1-\sigma)(1-\theta)}\theta\omega}{\frac{\omega}{1-\omega}\left(\frac{1+i_t}{\Pi_{t+1}}\left(f_{qt}+\nu_t\right) - \frac{f_{qt}}{\Pi_{t+1}}\right)} \right)^{\frac{1}{(1-\theta)+\theta\sigma}} \end{aligned}$$

where

$$\Omega_t \equiv \frac{\omega}{1-\omega}\left(1 - \frac{1}{1+i_t}\frac{f_{qt}}{f_{qt}+\nu_t}\right) .$$

Then one can iterate on the secured borrowing constraint (5) to determine ν_t.

Binding non negativity conditions Inada conditions on u ensure that non negativity constraints on m and d are never binding. However, in practice we choose very small but positive numbers as lower bounds for m and d. We make these sufficiently small so that the lower bounds would never bind before the collateral constraint. Since q_1 is chosen to reflect the lower bounds for a, m, and d, when $q_{t+1} = q_1$ at least one of the lower bounds will be binding.

Again $q_{t+1} = q_1$ requires that d_t and m_t be chosen at their lower bounds. In this case at least one of the non negativity constraints must bind. In all other cases, if m_0 and d_0 are chosen sufficiently small, Inada conditions should keep these constraints from binding.

We implicitly assume that the constraint on money binds first, in which case we can look for a solution where $\kappa > 0$ and $\chi =\geq 0$ (constraint on d is just binding).[22] In this event $d_t = d_0$ and $m_t = m_0$ and

$$\nu_t(m) = \frac{-f_q\left(q_t, d_t(m), z_t\right)\left(\frac{1+i_t}{\Pi_{t+1}}\left(1+\Psi_1\left(d_t(m), d_{t-1}\right)\right) - (1-\delta_D)\right) + f_d\left(q_t, d_t(m), z_t\right)}{\left(\frac{1+i_t}{\Pi_{t+1}}\left(1+\Psi_1\left(d_t(m), d_{t-1}\right)\right) - \mu(1-\delta_D)\right)} . \quad (39)$$

determines ν. Then use (33) to find a fixed point equation for c_t

$$c_t = \left(\left(\omega c_t^{\frac{\eta-1}{\eta}} + (1-\omega) m_0^{\frac{\eta-1}{\eta}}\right)^{\frac{1-\eta+\eta\theta-\eta\theta\sigma}{\eta-1}} d_{t-1}^{(1-\theta)(1-\sigma)} \frac{\theta\omega}{\left(f_{qt}+\nu_t\right)\frac{1+i_t}{\Pi_{t+1}}} \right)^{\eta}$$

with d_t, c_t, m_t and q_{t+1} then a_t follows from the law of motion (9) and q_t follows from the budget constraint (8).

C.1.4 Cashless Economy

In a cashless economy where $\omega \to 1$ little changes, but the solution is much simpler.

[22] In practice it could be the opposite, but we verify that the candidate solution does not violate any of the KKT conditions.

CES Preferences We use the parametric specification

$$u\left(c_t, d_{t-1}\right) \equiv \frac{1}{1-\sigma}\left({c_t}^{\theta} d_{t-1}^{1-\theta}\right)^{(1-\sigma)}$$

so

$$\begin{aligned}
u_{ct} &\equiv \left({c_t}^{\theta} d_{t-1}^{1-\theta}\right)^{-\sigma} c_t^{\theta-1} d_{t-1}^{1-\theta}\theta \\
u_{dt} &\equiv \left({c_t}^{\theta} d_{t-1}^{1-\theta}\right)^{-\sigma} c_t^{\theta} d_{t-1}^{-\theta}\left(1-\theta\right) \ .
\end{aligned}$$

Now we can write equation (27) as

$$\frac{f_{qt}+\nu_t}{\Pi_{t+1}} = \frac{\left({c_t}^{\theta} d_{t-1}^{1-\theta}\right)^{-\sigma} c_t^{\theta-1} d_{t-1}^{1-\theta}\theta}{1+i_t} \tag{40}$$

Interior solution With $\nu_t = 0$ then given f_{qt}

$$c_t = \left(\frac{\theta d_t^{(1-\sigma)(1-\theta)}}{f_{qt}\frac{1+i_t}{\Pi_{t+1}}}\right)^{\frac{1}{1-\theta(1-\sigma)}}$$

Binding collateral constraint We know d_t is within

$$d_t \in \left[d_0, \frac{q_{t+1}+b}{(1-\delta)(1-\mu)}\right]$$

so if d_t exceeds the upper bound then we set $d_t = \bar{d}_t$determine $\nu_t \geq 0$ as before.

C.2 Numerical Solution

First, define exogenous grid $D = \{d_1, \ldots, d_{N_d}\}$ (which will be the lagged values of d) and then define exogenous grid $Q = \left\{q_1, \ldots q_{N_q}\right\}$ where $q_1 = -b + d_0\left(1-\mu\right)\left(1-\delta_D\right) + \frac{m_0}{\Pi^*}$. The policy functions will be defined over a $S \times Q \times D$ grid.

We describe the solution with backwards induction in preparation for the non stationary solution. Of course letting $T \to \infty$ with constant prices we find the stationary policy rules. To find $f_{qt}\left(q_{t+1}, d_t, z_t\right)$ and $f_{dt}\left(q_{t+1}, d_t, z_t\right)$ we start with an initial guess for $m_{t+1}\left(q_{t+1}, d_t, z_{t+1}\right)$ and $c_{t+1}\left(q_{t+1}, d_t, z_{t+1}\right)$, and then use the envelope conditions (31) and (32) and the transition matrix P_s for z to compute $f_{qt}\left(q_{t+1}, d_t, z_t\right)$ and $f_{dt}\left(q_{t+1}, d_t, z_t\right)$ over the exogenous grid. Then for every $q \times z$ solve (30) for $d_t^N\left(q_{t+1}, z_t\right)$. Solve for the adjustment costs

$$\Psi(d_t^N\left(q_{t+1}, z_t\right), d_{t-1}) = \frac{\rho}{2}\left(\frac{d_t^N - (1-\delta_d)d_{t-1}}{d_{t-1}}\right)^2 d_{t-1} \text{ and } \Psi_1\left(d_t^N\left(q_{t+1}, z_t\right), d_{t-1}\right) = \rho\left(\frac{d_t^N}{d_{t-1}} - (1-\delta_d)\right)$$

Next using $f_{qt}\left(q_{t+1}, d_t^N, z_t\right)$ and $d_t^N\left(q_{t+1}, z_t\right)$ look for an interior solution for $c_t^N\left(q_{t+1}, d_{t-1}, z_t\right)$

using (36) and

$$\begin{aligned}
m_t^N\left(q_{t+1}, d_{t-1}, z_t\right) &= \left(\frac{1+i_t}{i_t}\frac{1-\omega}{\omega}\right)^{\eta} c^N \\
a_t^N\left(q_{t+1}, d_{t-1}, z_t\right) &= \Pi_{t+1}\left(q_{t+1} - d_t^N\left(1-\delta_D\right)\right) - m_t^N \\
q_t^N\left(q_{t+1}, d_{t-1}, z_t\right) &= c_t^N + \frac{a_t^N}{1+i_t} + d_t^N\left(q_t, e_t\right) + \Psi\left(d_t^N\left(q_t, z_t\right), d_{t-1}\right) \\
&\quad + m_t^N - w_t z_t\left(1-\tau_t\right) - T_t
\end{aligned}$$

Check the borrowing constraint Next check whether the interior solution for a^N would violate the collateral constraint (or check d^N). If

$$\frac{a_t^N\left(q_{t+1}, d_{t-1}, z_t\right)}{\Pi_{t+1}} < -\mu\left(1-\delta_D\right) d^N\left(q_{t+1}, z_t\right)$$

then the collateral constraint binds so that $\nu > 0$. When $\nu > 0$ we consider two cases, for all $q_{t+1} > q_1$ we look for a solution where $\nu > 0$ but the non negativity constraints are slack. For $q_{t+1} = q_1$ we look for a solution for at least one of the non negativity constraints binds as described above. When the collateral constraint binds and $d_t^N(q_t, z_t)$ is updated, the adjustment cost, Ψ_t, and the multiplier on the borrowing constraint, ν_t, will also need to be updated

Interpolating over the exogenous grid Finally, using the policy functions defined over the endogenous grid, interpolate over the exogenous Q gridpoints, and correct for any $q < q^N\left(q_1, d, z\right)$, which requires that the collateral, money, and durable constraints all bind (the Inada conditions should prevent this if m_0 and d_0 are sufficiently small). In which case, policy can be determined directly from the constraints. Use the policy functions $c\left(q, d, z\right)$ and $m\left(q, d, z\right)$ over the exogenous grid $Q \times D \times S$ to update f_q and f_d via the envelope conditions. Iterate until the policy functions converge.

D Path of aggregate variables

In Figure D.1 we compute the response of the aggregate variables along the transition path between the high and low inflation steady states, which is plotted as the solid line. The star represents the level of the aggregate variables in period 0's high inflation equilibrium and the broken line indicates the new low inflation long-run values. The reduction in inflation induces an economy wide portfolio rebalancing. In the model with money the portfolio rebalancing also creates a one time jump in the level of real balances as the central bank responds to households' additional demand for liquid assets. Although this outcome may be surprising, we see a similar movement in 1983 when, empirically, household expectations of lower inflation seemed to first stick.

The path of the aggregate variables changes significantly between the models with and without money. In the model with money, plotted in panel (a), indebted households find themselves with more debt than anticipated and wish to deleverage. At the same time, all households wish to rebalance their assets away from equity and durables towards money and consumption which has become cheaper with the lower inflation tax. This causes an initial decrease in aggregate investment and output while aggregate real balances and consumption rise to their new steady state level. As households reach their optimal portfolios, investment and output begin to recover to their new steady state levels.

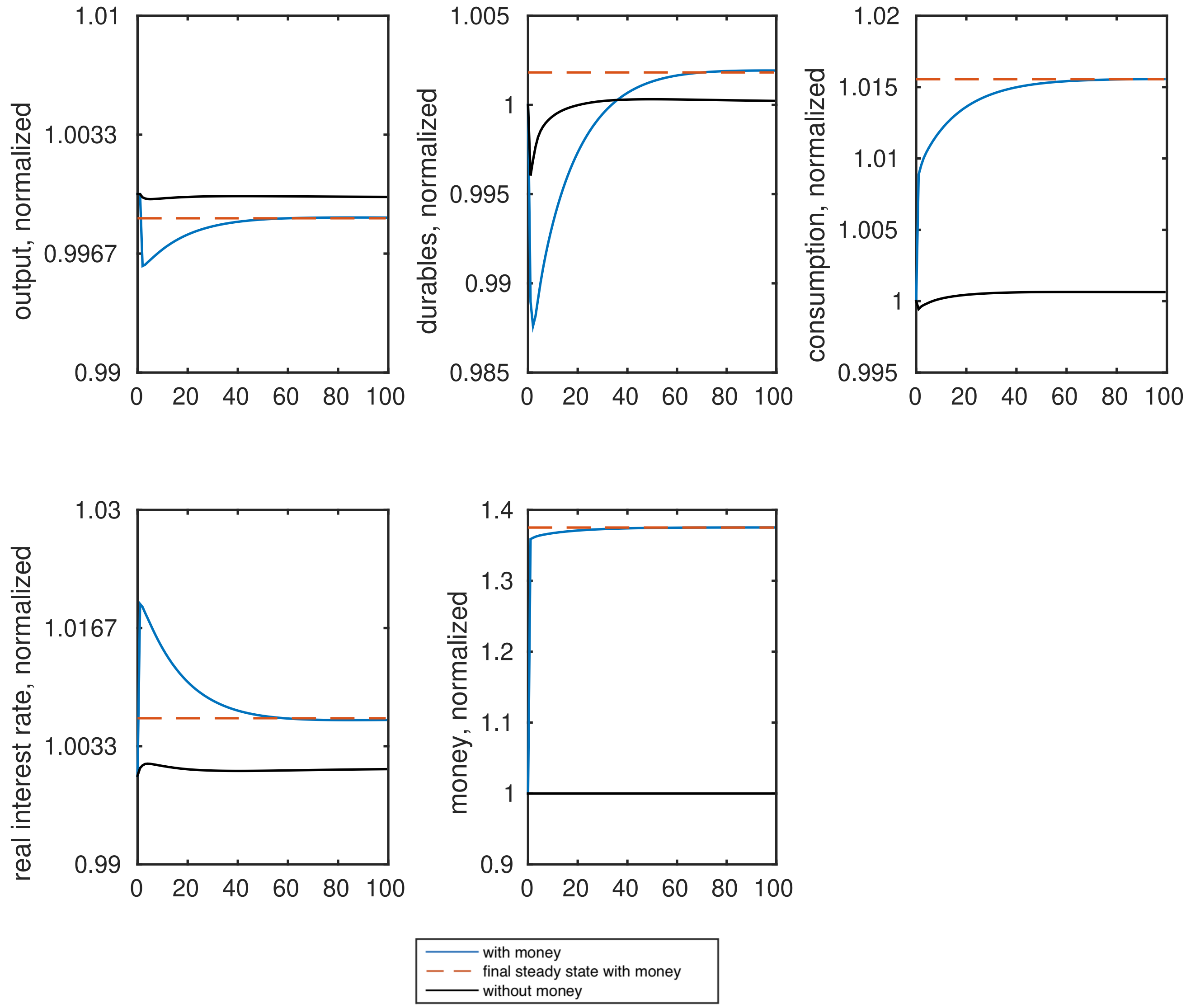

Figure D.1: Path of aggregate variables

Note: figure shows the path of aggregate variables in the model with and without money. All variables are normalized relative to the initial steady state. The blue line shows the path of aggregate variables in the model with money and the red dashed line shows the aggregate variables in final steady state in the model with money. The black line shows the path of the aggregate variables in the model with out money. Without money, monetary policy is super-neutral so the path of aggregate variables will converge back to the initial steady state, which is normalized to be one.

In the model without money, plotted in panel (b), the change in inflation is a pure redistribution of wealth. The path of the aggregate variables reflects the balance between the response of the winners and losers from the redistribution which are largely offsetting. Thus, on impact, aggregate consumption declines, but it will increase above its long run level before returning to steady state. This is because the nominal borrowers who lose from the redistribution have a much higher marginal propensity to consume out of additional wealth. They deleverage quickly and increase their consumption back to its equilibrium level. Initial savers, on the other hand, have a lower marginal propensity to consume. In response to the windfall they receive from the redistribution, they increase their consumption slightly but maintain the higher level for a longer period of time. Auclert (2015) emphasizes the ability of the central bank to stimulate aggregate consumption by an inflation which redistributes wealth from savers to borrowers. This channel holds in the model without money, but is reversed by the decline in the inflation tax. As the inflation tax declines, consumption goods become cheaper for all households which is enough to reverse the losses from the redistribution for most households.

Figure D.1 also plots the paths of the real interest rate in the models with and without money. When money is valued for liquidity services, the real interest rate jumps in response the to the change in steady state inflation. The price of holding money, and therefore of consumption, is lower in the low inflation steady state. As a result, households decrease their saving in order to consume and hold more cash. In response, the real interest rate jumps in order to encourage households to save more which is necessary for markets to clear. After the initial jump, as households reach their optimal portfolio, the real interest rate slowly decreases to its new steady state level. However, in this model, money is not super-neutral. The real interest rate in the low inflation steady state is higher than the high inflation steady state since households have decreased their savings in favor of money and consumption.

In the model with no money, the shock to steady state inflation causes the real interest rate to decrease on impact. The inflation shock results in a redistribution of wealth from borrowers to savers. As savers find themselves wealthier than their ideal they increase their supply of savings. Borrowers, on the other hand, become more indebted than they intended. Over time they deleverage decreasing their consumption and increasing their savings. The overall result is a small drop in the real interest rate as demand for nominal debt goes down which then slowly returns to its steady state level. Without money, the change in the inflation rate is super neutral so the real interest eventually returns to its previous level.